

An Observational Test of Holographic Inflation

James E. Lidsey and David Seery

*Astronomy Unit, School of Mathematical Sciences, Queen Mary,
University of London, Mile End Road, London, E1 4NS, UK*

Observational consequences of inflationary cosmology in the holographic dual of the Randall-Sundrum type II braneworld scenario, as motivated by the AdS/CFT correspondence, are investigated. High energy corrections to the standard four-dimensional Friedmann equation induce a corresponding modification to the form of the single-field inflationary consistency equation based on Einstein gravity. The degree of departure from the standard expression is determined by the ratio, r , of the primordial tensor and scalar perturbation amplitudes and the coefficient, c , of the conformal anomaly in the dual gauge theory. It is found that a necessary condition for detecting such a correction with the next generation of cosmic microwave background (CMB) polarization experiments is that $r \geq 0.06$. The bound is tightened to $r > 0.3$ for values of the central charge that are compatible with known compactifications of type IIB string theory as parametrized in terms of F-theory compactification on Calabi-Yau four-folds. This is close to the present upper bounds inferred from combined observations of the CMB anisotropy power spectrum and high redshift surveys. We conclude that if such modifications to the inflationary consistency equation are to be observable, the gravitational wave background should be detected in the near future. A further consequence of the non-standard dynamics at high energies is that the initial state of the universe is a quiescent singularity with a finite density and pressure.

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I. INTRODUCTION

The AdS/CFT correspondence represents one of the most striking insights to have emerged from string/M-theory in the past decade [1]. (For a review, see, [2]). This correspondence implies that a gravitational theory on $(d+1)$ -dimensional Anti-de Sitter (AdS_{d+1}) space admits a dual description in terms of a conformal field theory (CFT) propagating on the d -dimensional boundary.

On the other hand, recent years have witnessed tremendous advances in observational cosmology with the high-precision data announced by the Wilkinson Microwave Anisotropy Probe (WMAP) and other Cosmic Microwave Background (CMB) surveys strongly supporting the core predictions of inflationary cosmology, namely a spatially-flat universe with a primordial spectrum of adiabatic, gaussian and nearly scale-invariant density perturbations [3, 4].

In view of these developments, there is a pressing need to understand how inflation may be incorporated within string/M-theory and to confront the predictions of the models with present-day and forthcoming cosmological observations. Motivated by developments in string/M-theory, the braneworld paradigm – where our observable universe is viewed as a co-dimension one brane propagating in a higher-dimensional ‘bulk’ space – has received considerable attention. (See [5] for reviews). Of particular interest is the Randall-Sundrum type II (RSII) scenario, where the brane is embedded in an Einstein space sourced by a negative cosmological constant [6]. The effective localization of the graviton zero-mode on the brane can be understood within the AdS/CFT context by interpreting the model as a cut-off, strongly cou-

pled conformal gauge theory coupled to four-dimensional Einstein gravity [7, 8]. This holographic dual formulation of the RSII scenario can be viewed as a four-dimensional theory in its own right with an effective action comprised of a sum of contributions, $S_{\text{dual}} = S_{\text{loc}} + \Gamma + S_{\text{mat}}$, where S_{loc} represents the Einstein-Hilbert action (including a possible cosmological constant), S_{mat} represents standard matter fields and Γ is the action for the conformal anomaly of the gauge theory [7, 8, 9, 10, 11, 12]. In the spatially isotropic Friedmann-Robertson-Walker (FRW) cosmology, such a dual interpretation modifies the form of the Friedmann equation away from the standard $H^2 \propto \rho$ dependence on the energy density.

Here we investigate whether such modifications to the cosmic dynamics may leave an observable signature in the CMB. As well as providing a causal mechanism for generating the primordial spectrum of density perturbations, inflation also produces a spectrum of tensor (gravitational wave) fluctuations [13]. A direct detection of the amplitude and tilt of such a spectrum would yield crucial information on the energy scale of inflation. Moreover, in conventional single-field slow-roll inflation, the amplitudes of the perturbation spectra, $A_{S,T}^2$, are related to the tensorial tilt, n_T , through a ‘consistency’ equation that is independent of the form of the inflaton potential [34]:

$$\frac{A_T^2}{A_S^2} = -\frac{1}{2}n_T. \quad (1)$$

Standard slow-roll inflation driven by a single inflaton field could therefore be verified or ruled out by confronting the relation (1) with observations. Eq. (1) also arises as a consistency equation in a number of braneworld inflationary scenarios [15].

In general, modifications to the Friedmann equation such as those arising in the holographic dual of the RSII scenario will induce departures from the standard consistency equation (1). We determine the prospects for detecting such a departure in future CMB polarization experiments. Polarization of the CMB represents the most promising route toward a direct detection of the gravitational wave background, since the divergence-free B -mode of the polarization anisotropy can only be produced (at leading-order) by tensor perturbations [16]. Consequently, considerable attention has focused recently on understanding the experimental and theoretical issues involved in detecting a primordial B -mode polarization and a number of experiments are planned for the near future. (See [17] for a review). For example, the Planck surveyor [18] will detect a scalar-to-tensor ratio above $r \approx 0.02$, while Clover [19] will be sensitive to values as small as $r \approx 0.06$. We find that a departure from the standard inflationary consistency equation could in principle be detectable if the amplitude of the gravitational wave spectrum exceeds $r \approx 0.06$.

II. HOLOGRAPHIC INFLATION

We begin by summarizing the derivation of the modified form of the Friedmann equation in the holographic dual of the RSII scenario [35]. The dual four-dimensional action can be calculated from the holographic renormalization group approach developed by de Boer, Verlinde and Verlinde [12]. This is based on the Hamilton-Jacobi (HJ) equation of General Relativity [20]. In this prescription, the fourth-order contribution to the generating functional of the HJ equation is identified as the effective action, Γ , for the gauge theory. The dual action then takes the form $S_{\text{dual}} = S_{\text{loc}} + \Gamma + S_{\text{mat}}$, where S_{loc} contains terms that are no higher than second-order in derivatives and includes the Einstein action that renders the bulk AdS action finite. (We will assume implicitly that the tension of the brane is tuned to cancel the local (divergent) contribution arising from an effective cosmological constant). The matter fields on the brane are described by the action S_{mat} . Extremizing the variation of the dual action with respect to the metric tensor yields the gravitational field equations, $\hat{m}_P^2 G_{\mu\nu} = T_{\mu\nu} + V_{\mu\nu}$ where $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu} = 2(-g)^{-1/2} \delta S_{\text{mat}} / \delta g^{\mu\nu}$ is the energy-momentum of the ordinary matter sector, $V_{\mu\nu} \equiv 2(-g)^{-1/2} \delta \Gamma / \delta g^{\mu\nu}$ is the effective stress tensor of the CFT and $\hat{m}_P = m_P / \sqrt{8\pi}$ is the reduced Planck mass.

The specific form of the trace of the CFT energy-momentum tensor follows directly from the fourth-order HJ equation and is given by [12, 20, 21]

$$V^\mu{}_\mu = \frac{\ell^3 M^3}{8} \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right), \quad (2)$$

where ℓ is the AdS radius of curvature and is determined in terms of the five-dimensional bulk cosmological con-

stant, Λ , such that $\Lambda = -12M^3/\ell^2$, where M is the five-dimensional Planck scale.

On the other hand, it is also known from direct calculation in four dimensions [11] that the trace anomaly has the form $V^\mu{}_\mu = -\tilde{a} C_{\mu\nu\lambda\rho} C^{\mu\nu\lambda\rho} - cG$, where $C_{\mu\nu\lambda\rho}$ is the Weyl tensor, G is the Gauss-Bonnet combination of curvature invariants and the constants $\{\tilde{a}, c\}$ are determined by the field content of the CFT such that

$$\begin{aligned} \tilde{a} &= -\frac{1}{120(4\pi)^2} (N_S + 6N_F + 12N_V), \\ c &= \frac{1}{360(4\pi)^2} (N_S + 11N_F + 62N_V), \end{aligned} \quad (3)$$

where N_I denotes the number of scalar, fermionic and vector degrees of freedom, respectively, and the fermions are Dirac fermions. The field content of Yang-Mills theory yields $N_S = 6N^2$, $N_F = 2N^2$ and $N_V = N^2$ for $N \gg 1$, where N denotes the number of colours in the gauge group. This implies that $c = (N/8\pi)^2$ and $\tilde{a} = -c$. Hence, equating the above expression for the trace with Eq. (2) relates the conformal anomaly coefficient directly to the AdS radius, $c = (M\ell/2)^3$.

For a spatially flat FRW line element with scale factor a , we may define $V_{00} \equiv \sigma$ and $V_{ij} \equiv \sigma_P a^2 \delta_{ij}$. The Bianchi identity, together with conservation of the energy-momentum, $T^{\mu\nu}{}_{;\mu} = 0$, then implies that $V^{\mu\nu}{}_{;\mu} = 0$, i.e., that

$$\dot{\sigma} = -3H(\sigma + \sigma_P), \quad (4)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter. On the other hand, the trace anomaly (2) simplifies to

$$\sigma - 3\sigma_P = 24c \frac{\ddot{a}}{a} \frac{\dot{a}^2}{a^2} \quad (5)$$

and substituting Eq. (5) into Eq. (4) yields, up on integration, the solution $\sigma = \chi + 6cH^4$, where $\chi \propto 1/a^4$ is an effective radiation term. During inflation, this term is rapidly redshifted away and we therefore neglect its contribution. The (00)-component of the field equations then yields the effective Friedmann equation

$$H^2 = \frac{\hat{m}_P^2}{4c} \left[1 + \epsilon \sqrt{1 - \frac{8c}{3\hat{m}_P^4} \rho} \right], \quad (6)$$

where $\epsilon = \pm 1$. Note that there exists an *upper bound* to the energy density, $\rho < \rho_{\text{max}} = 3\hat{m}_P^4/(8c)$. The $\epsilon = -1$ branch reduces to the correct form of the Friedmann equation in the low-energy limit, $H^2 \propto \rho$, and we therefore focus on this case in the following.

We will assume the dynamics of the universe is driven by a single scalar field with self-interaction potential $V(\phi)$. Covariant conservation of its energy-momentum tensor therefore implies that

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \quad (7)$$

Eq. (7) may also be expressed in the form $\dot{\rho} = -3H\dot{\phi}^2$, where $\rho = \dot{\phi}^2/2 + V$ represents the energy density of the field.

It proves convenient to define a new variable, θ :

$$\rho \equiv \frac{3\hat{m}_P^4}{8c} \cos^2 \theta, \quad (8)$$

where $\theta \in [0, \pi/2]$, and an equation of state parameter, $\gamma \equiv \dot{\phi}^2/\rho$. Differentiating Eq. (6) then implies that

$$\dot{H} = -\frac{1}{2\hat{m}_P^2} \frac{\gamma\rho}{\sin \theta}. \quad (9)$$

The necessary and sufficient condition for inflation, $\dot{H} + H^2 > 0$, may then be expressed as a lower limit on the value of θ for a given equation of state parameter:

$$\sin \theta > \frac{3\gamma}{4-3\gamma}. \quad (10)$$

For a semi-positive-definite self-interaction potential, $V \geq 0$, the equation of state is bounded, $0 \leq \gamma \leq 2$. As the energy density approaches the upper bound $\rho \rightarrow \rho_{\max}$ ($\theta \rightarrow 0$), it follows that the pressure, Hubble parameter and scale factor of the universe also remain finite. However, Eq. (9) implies that the universe has infinite deceleration at this point, $\ddot{a} \rightarrow -\infty$. The model is therefore geodesically incomplete, since the Ricci scalar diverges. On the other hand, this does not represent a conventional big bang singularity since the energy density remains finite. Such a past ‘quiescent’ curvature singularity was also recently uncovered in a particular class of five-dimensional braneworld models [22]. It is similar to the sudden future singularities discussed recently by Barrow [23], although the pressure of the fluid is also divergent in Barrow’s cases. In effect, the existence of this quiescent singularity delays the onset of inflationary expansion. Indeed, a necessary (but not sufficient) condition for inflation is that $\gamma < 2/3$, whereas this condition is necessary *and* sufficient in the standard scenario.

We may view Eqs. (6)–(7) as an effective four-dimensional cosmology. Conservation of energy-momentum, Eq. (7), then implies that the adiabatic curvature perturbation on a uniform density hypersurface, $\zeta = H\delta\phi/\dot{\phi}$, is conserved on large scales [24]. (We assume implicitly throughout that scales that are observable today first crossed the Hubble radius during a phase of slow-roll inflation, where $|\dot{H}|/H^2 \ll 1$ and $|\ddot{\phi}| \ll H|\dot{\phi}|$.) As a result, the amplitude of a perturbation mode when it re-enters the Hubble radius after inflation is given by $A_S^2 = 4\langle\zeta^2\rangle/25$, where the right-hand side is evaluated when the mode first crosses the Hubble radius during inflation, i.e., when $k = aH$, where k is the comoving wavenumber. The scalar field fluctuation at this epoch is determined by the Gibbons-Hawking temperature of de Sitter space, $\langle\delta\phi\rangle = H^2/(4\pi^2)$, and the scalar perturbation amplitude is therefore given by

$$A_S^2 = \frac{1}{25\pi^2} \frac{H^4}{\dot{\phi}^2}. \quad (11)$$

The tensor and matter perturbations decouple from one another to first-order. The large-scale amplitude of each tensor mode when crossing the Hubble radius during inflation is therefore determined solely by the expansion rate of the universe, as in conventional slow-roll inflation [13]. This implies that

$$A_T^2 = \frac{4}{25\pi} \frac{H^2}{m_P^2} = \frac{1}{200\pi^2 c} (1 - \sin \theta). \quad (12)$$

The tilt of the tensor perturbation spectrum, $n_T \equiv d \ln A_T^2 / d \ln k$, is then determined by differentiating Eq. (12) with respect to comoving wavenumber and substituting in Eqs. (11) and (12). We find that

$$n_T = -2 \frac{A_T^2}{A_S^2} \frac{1}{\sin \theta}. \quad (13)$$

Comparison between Eqs. (1) and (13) implies that the standard inflationary consistency equation is modified due to the corrections arising in the Friedmann equation (6). It is still the case that $n_T \leq 0$, implying immediately that this holographic scenario could be ruled out by a detection of a positive spectral index. More specifically, however, we deduce that for a given scalar-to-tensor ratio, the magnitude of the spectral index is *enhanced* by a factor of $\sin \theta$ relative to the standard scenario.

III. HOLOGRAPHIC CONSISTENCY RELATION

Eq. (13) can be rearranged into a more convenient form by substituting in Eq. (12):

$$n_T + 2 \frac{A_T^2}{A_S^2} = -\frac{400\pi^2 c}{1 - 200\pi^2 c A_T^2} \frac{A_T^4}{A_S^2} \quad (14)$$

and Eq. (14) can be rewritten in terms of observable parameters by relating the perturbation amplitudes $A_{S,T}^2$ directly to the corresponding quadrupole variances in the power spectrum of the CMB, $C_2^{S,T}$. We consider a (spatially flat) concordance cosmology with a dark energy density parameter $\Omega_\Lambda = 0.66$ and reduced Hubble parameter $h = 0.66$. The observable tensor-to-scalar ratio is then defined by $r \equiv T/S$, where $T \equiv 5C_2^T/4\pi = 1.4f_T A_T^2$ and $S \equiv 5C_2^S/4\pi = 0.1f_S A_S^2$, and the ‘transfer functions’ are defined by $f_S = 1.04 - 0.82\Omega_\Lambda + 2\Omega_\Lambda^2 \approx 1.37$ and $f_T = 1.0 - 0.03\Omega_\Lambda - 0.1\Omega_\Lambda^2 \approx 0.94$, respectively [25]. These definitions take into account the non-negligible contribution to the quadrupole from the late-time integrated Sachs-Wolfe effect in a dark energy dominated universe. The COBE normalization of the temperature anisotropy power spectrum is $S = 5.5 \times 10^{-11}$ [26] and it follows that $r = T/S = 9.6A_T^2/A_S^2 = 1.8 \times 10^{10}T$. Hence, the consistency equation (14) may be written in the form

$$n_T + \frac{r}{4.8} = -\frac{1}{4.8} r r_l \quad (15)$$

where

$$r_l \equiv \frac{\tilde{c}r}{1 - \tilde{c}r}, \quad \tilde{c} \equiv 8.25 \times 10^{-8}c. \quad (16)$$

Since Eq. (15) is independent of the specific functional form of the inflaton potential, it may be interpreted as the consistency equation for single field inflation in the holographic dual of the RSII braneworld cosmology. For a given set of observations, the only free parameter in this relation is the coefficient of the conformal anomaly, c , or equivalently, the number of colours in the dual gauge theory. Since a departure from the standard consistency equation is parametrized by a non-zero right-hand side in Eq. (15), we will refer to r_l as the ‘departure parameter’.

The question that now arises, therefore, is whether the holographic consistency equation can be observed. Song and Knox (SK) [27] have discussed the prospects of detecting a departure from the standard consistency equation that has precisely the form (15). In SK, however, r_l is viewed as a free parameter that arises from the inclusion of quantum loop corrections to the two-point correlation function of the inflaton [36]. Although the physical interpretation of the departure parameter is different in the present context, we may employ the results of SK to investigate the detectability prospects of the holographic consistency equation. SK consider a future CMB polarization experiment with full sky coverage, an angular resolution in the range $1.0' \leq \varphi \leq 30.0'$, and a noise level in the range $1\mu\text{K} \cdot \text{arcmin} < \Delta_T < 15\mu\text{K} \cdot \text{arcmin}$, where $\Delta_T = 1/\sqrt{2\omega}$ and ω is the weight per solid angle. They further assume an optical depth parameter $\tau = 0.17$ and proceed to calculate the anticipated error on $n_T + r/4.8$ as a function of the tensor-to-scalar ratio r . The results are summarized in Table I for a B -mode that has been ‘cleaned’ by estimating the projected lensing potential from the four-point function of the temperature and polarization fields [28].

In practice, this implies that for a given fiducial value of r , one may determine how large the departure parameter r_l must be for the modification in the holographic consistency equation to be observable. A necessary condition is that the right-hand side of Eq. (15), $Q \equiv rr_l/4.8$, should exceed σ_T , the anticipated error in $n_T + r/4.8$. The results are shown in Fig. 1 and the third column of Table I. The lower limit on r_l increases monotonically for decreasing r .

SK regard r_l as a free parameter of the theory rather than an observable. However, in the holographic consistency relation, the departure parameter depends on the tensor-to-scalar ratio and will therefore be subject to experimental errors. Although it is to be expected that the most significant errors will arise from measurements of the tensorial tilt, the error in the departure parameter will limit how low the observed value of r can be if a deviation is to be observable. The error in $Q \equiv rr_l/4.8$ (the right-hand side of the holographic consistency equation (15)) is estimated to be $|\delta Q| = r_l(2 + r_l)|\delta r|/4.8$ and, if a non-zero value of Q is to be detectable, a necessary condition is that $|\delta Q| < Q$. Hence, given a fiducial value of the

r	σ_T	$r_{l,\text{lower}}$	c_{min}	N_{min}
0.71	0.012	0.08	1.3×10^6	2.8×10^4
0.63	0.012	0.09	1.6×10^6	3.2×10^4
0.51	0.012	0.12	2.5×10^6	4.0×10^4
0.40	0.013	0.15	4.0×10^6	5.0×10^4
0.32	0.014	0.20	6.3×10^6	6.3×10^4
0.21	0.016	0.36	1.6×10^7	1.0×10^5
0.10	0.020	0.92	5.6×10^7	1.9×10^5
0.07	0.023	1.47	9.7×10^7	2.5×10^5
0.06	0.025	2.00	1.4×10^8	2.9×10^5
0.05	0.027	2.73	1.9×10^8	3.4×10^5
0.02	0.036	9.01	5.7×10^8	6.0×10^5

TABLE I: Summarizing the prospects for detecting the holographic consistency equation (15). The second column represents the expected error on $n_T + r/4.8$, denoted by σ_T , for fiducial values of the tensor-to-scalar ratio, r , in the future CMB polarization experiment of SK [27]. Angular resolution is specified to be $\varphi = 1'$ with a noise level $\Delta_T = 3\mu\text{K} \cdot \text{arcmin}$. The third column $r_{l,\text{lower}}$ denotes the minimum value of the departure parameter that will lead to an observable departure from the standard consistency equation. The fourth and fifth columns represent the corresponding limits on the conformal anomaly coefficient, c , and the number of colours in the gauge theory, N , respectively. The horizontal line at $r \approx 0.06$ indicates the critical value of r where the projected observable errors on $Q \equiv rr_l/4.8$ are comparable to the magnitude of Q itself given that $r_l > r_{l,\text{lower}}$ (see the text for details). Detection of the holographic consistency equation is only possible for $r \geq 0.06$.

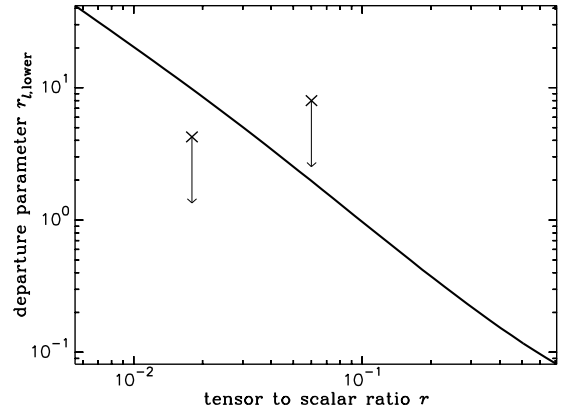


FIG. 1: Illustrating the minimal value of the departure parameter r_l that will lead to a detection of the holographic consistency equation by the future full-sky CMB polarization experiment considered in SK [27]. In principle, for a given fiducial value of the tensor-to-scalar ratio, r , a detection is possible in the region of parameter space above the solid line. The crosses denote points corresponding to the future full-sky polarization experiment considered by VPJ [29], where the errors on r saturate the upper limit (17). For these values of r , departures from the standard inflationary consistency equation will only be detectable for values of r_l smaller than those indicated by the crosses. Hence, compatibility between the upper and lower bounds requires $r \geq 0.06$.

tensor-to-scalar ratio with an associated error, this constraint may be interpreted as an *upper limit* on the value of the departure parameter that will lead to a detection, i.e., we require that

$$r_l < \frac{r}{|\delta r|} - 2. \quad (17)$$

Consequently, by combining this constraint with the lower limit $r_{l,\text{lower}}$ shown in Fig. 1, we may estimate the smallest allowed value of r that will lead (in principle) to a detection of the holographic consistency equation.

Recently, Verde, Peiris and Jimenez (VPJ) have quantified the level of accuracy at which a primordial gravitational wave background could be detected using a variety of space- and ground-based polarization experiments [29]. (See also [30]). Specifically, they have determined the anticipated errors in the tensor-to-scalar ratio for fiducial values of r *without* imposing the standard consistency equation. (In general, fixing n_T through Eq. (1) greatly reduces the error in r). For the case of a realistic future all-sky satellite experiment with no foreground subtraction or delensing, VPJ conclude that an accuracy of $|\delta r| = 0.003$ when $r = 0.02$ is possible, while a ratio of $r = 0.06$ could be detected with an accuracy of $|\delta r| = 0.006$. They also find that the constraints on $|\delta r|$ do not alter significantly even when foreground contamination is reduced to 1% of its original level or when delensing techniques are applied [29]. This level of accuracy at $r = 0.02$ implies that $r_l < 5$ is required for a detection of Eq. (15) and this is inconsistent with the results of Fig. 1, as illustrated by the cross in the diagram. On the other hand, if $r = 0.06$, we would require $r_l < 8$ and this is compatible with Fig. 1. We may conclude, therefore, that a future detection of the holographic consistency equation will require the tensor-to-scalar ratio to satisfy $r \geq 0.06$. The existence of such a limit follows since the modifications to the standard consistency equation become smaller at lower energies, corresponding to $\sin \theta \rightarrow 1$ in Eq. (13).

IV. EXPECTATIONS FROM ADS/CFT

Thus far, we have considered empirical constraints that need to be satisfied for a detection of Eq. (15). However, the conformal anomaly coefficient, c , is related to the departure parameter through Eq. (16) and increases monotonically with r_l for a given value of r . Hence, the necessary condition on r_l for a detection of the holographic consistency equation may be interpreted as a necessary lower limit on the conformal anomaly coefficient, or equivalently, on the number of colours in the dual gauge theory. These limits are summarized in the fourth and fifth columns of Table I and Fig. 2 [37].

Such limits may be compared to what is expected from field theoretic considerations. In its simplest form, the AdS/CFT correspondence relates string theory on $AdS_5 \times S^5$ with N units of four-form flux to $\mathcal{N} = 4$

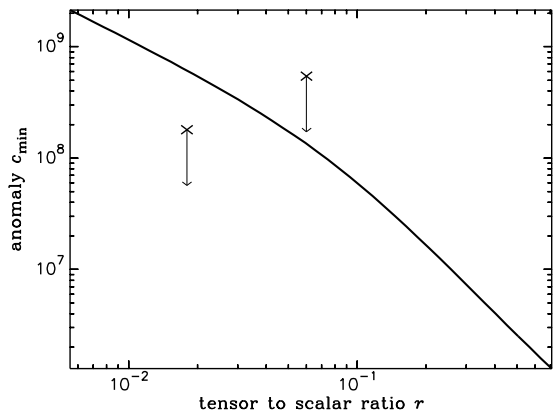


FIG. 2: As in Fig. 1, but illustrating the lower limit on the conformal anomaly coefficient that leads to a detectable departure from the standard inflationary consistency equation for a given observed tensor-to-scalar ratio.

SU(N) superconformal field theory [1]. For AdS_5 backgrounds arising from compactifications of the type IIB string theory that include D3-branes, N represents the number of branes that are present. In the majority of string compactifications that have been studied to date, $N \approx 10$ and this is clearly well below detectable levels. Nonetheless, higher values of N are possible [7, 31]. A powerful geometric way of describing compactifications of type IIB string theory is through F-theory compactification on Calabi-Yau four-folds, K_8 , that admit an elliptic fibration with a section, i.e., are locally the product $K_8 = K_6 \times T^2$, where K_6 is a complex three-fold [32, 33]. (The four-fold K_8 is not the physical compactified space, but provides a convenient way of parametrizing the geometry and moduli field VEVs).

Global tadpole cancellation implies that the effective total D3-brane charge is constrained by the topology of the K_8 such that $N_{D3} = \chi/24 - \mathcal{F}$, where $\chi(K_8)$ is the Euler characteristic of the four-fold and \mathcal{F} represents the number of fluxes present [32, 33]. Large values of N are therefore possible for suitable choices of K_8 . The topological properties of Calabi-Yau four-folds have been studied extensively [32, 33]. For the case of manifolds that are known explicitly (corresponding to the class of four-folds that can be represented as hypersurfaces in weighted projective spaces) the Euler number can be as high as $\chi \leq 1,820,448$ [33], thus allowing as many as $N_{D3} \approx \chi(K_8)/24 \approx 7.5 \times 10^4$ D3-branes to be present. This implies a maximal central charge of $c \approx 9 \times 10^6$ and comparing this with the minimal values presented in Table I indicates that the scalar-to-tensor ratio must exceed $r > 0.3$ if the holographic consistency equation is to be detectable. On the other hand, such a model would lack fluxes and would therefore probably not be realistic from a phenomenological point of view since the moduli fields would be effectively massless [38]. Consequently, a conservative (and more realistic) estimate for the maximal value of the D3-brane charge might be in the range

$N_{D3} \approx 1 - 5 \times 10^4$, although it should be noted that a definitive mechanism for moduli stabilization has yet to be proposed. It should also be emphasized that the above discussion applies only for known manifolds and examples admitting a larger D3-brane charge are not ruled out at the present time. In this sense, therefore, N may still be regarded as a free parameter.

Such a large hidden CFT sector with $10N^2 \approx 10^{11}$ degrees of freedom could be problematic for cosmology, and primordial nucleosynthesis in particular. However, the nucleosynthesis bounds can be satisfied if the standard model and CFT degrees of freedom are decoupled and satisfy the condition $\rho_{\text{CFT}} < \rho_{\text{CMB}}$ at the present epoch, where ρ_{CMB} is the energy density of the CMB. This ensures that $\rho_{\text{CFT}} < \rho_{\text{SM}}$ at energy scales ≈ 1 MeV, where a subscript ‘SM’ denotes standard model degrees of freedom. This bound may not necessarily be satisfied if energy leaks from the standard model into the CFT, but it can be shown on dimensional grounds [7] that this is not a problem if the parameter $\kappa \equiv \ell^2 T_{\text{SM}}^5 H^{-1} l_P^2$ is less than unity, where T_{SM} denotes the temperature. Recalling that $H \propto a^{-3/2}$ and $H \propto a^{-2}$ during the matter- and radiation-dominated phases of the universe’s history, respectively, and defining the redshift $z \equiv 1 + a_0/a$, then implies that

$$\ell \approx \frac{\kappa^{1/2}}{l_P T_0^{5/2} H_0^{-1/2} z_{\text{LS}}^{1/4}} \frac{1}{z^{3/2}} \approx 6 \times 10^{48} \left(\frac{\kappa}{z^3} \right)^{1/2} l_P \quad (18)$$

where in the second equality we have taken $T_0 = 2.4 \times 10^{-13} \text{ GeV}$. Gravity waves generated from standard, single-field inflation will be detectable from polarization of the CMB if the energy scale of inflation exceeds $3.2 \times 10^{15} \text{ GeV}$, corresponding to an inflationary redshift of $z_{\text{inf}} \approx 10^{28}$ [30]. Thus, requiring $\kappa < 1$ back to a redshift of z_{inf} then leads to an upper limit on the AdS_5 length scale such that $\ell < 6 \times 10^6 l_P$. In the AdS/CFT dictionary, $N \approx \ell/l_P$, so this is comfortably within the bound required for detectability of the holographic consistency equation.

V. CONCLUSION

At the present time, upper bounds on the scalar-to-tensor ratio are deduced from observations of the CMB anisotropy power spectrum and high redshift surveys. These bounds are sensitive to the priors assumed on the value and running of the scalar spectral index, n_S , as well

as the data sets included. The limit from the WMAP data alone, assuming no priors on $dn_S/d \ln k$ and n_S , is $r < 0.7$ at 95% confidence, but this strengthens to $r < 0.49$ if no running in n_S is assumed [3]. Combining CMB data sets with other large-scale structure surveys leads to the somewhat tighter bounds of $r < 0.36$ with no prior on the running and $r < 0.27$ with the prior $dn_S/d \ln k = 0$ assumed [4].

We have found that a necessary condition for an observable detection of the holographic consistency equation (15) is that $r \geq 0.06$. This is well within the level of sensitivity attainable with near-future CMB polarization experiments such as Planck [18] and Clover [19]. Within the context of the AdS/CFT correspondence, this lower bound transforms into a limit on the number of colours in the dual gauge theory, $N > 3 \times 10^5$. This is only a factor of 4 or so higher than the upper limit attainable in known compactifications of the type IIB string. Indeed, for values in the range $N \approx 6 \times 10^4$ or lower, a detection should be possible in principle if $r \geq 0.3$ and this is tantalizingly close to the current upper limits on the tensor-to-scalar ratio. This implies that a positive detection of the holographic consistency equation would place a strong limit on the central charge of the dual CFT in the range $N \approx 10^4 - 10^5$ and consequently on the topology of the higher dimensions.

On the other hand, we may take a more phenomenological view and interpret the Friedmann equation (6) as an independent four-dimensional model. In this case, a detection of the holographic consistency equation could be interpreted as observational evidence for modifications to Einstein gravity at very high energy scales. Moreover, if modifications to Einstein gravity of this nature are ever to be observable, we should expect to detect a gravitational wave background in the relatively near future. Conversely, observations that indicated $n_T + r/4.8 \neq 0$ and $r < 0.06$ would rule out the scenario (6). This offers the promise of constraining gravitational physics at energies inaccessible to any other form of experiment and the observations are eagerly awaited.

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- [34] We adopt the normalization conventions of Ref. [14]. We also define the observable tensor-to-scalar ratio in terms of the CMB quadrupole moments modified to account for a dark energy component [25]. This is related to the presently favoured definition used by the WMAP team such that $r_{\text{WMAP}} = (16/9.6)r = 1.67r$.
- [35] The reader is referred to [10, 12] for more detailed discussions.
- [36] We neglect the possible effects of such field theoretic corrections in the present work.
- [37] As a first approximation, we neglect any observational errors in r_l .
- [38] We thank E. Kiritsis for highlighting this point.